Faculty Innovator Grant 2014
Center for Learning and Teaching

Final Report Form

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Project Title: (10 words or less)	Developing interactive modules for 3D visualization in multivariable calculus

Other faculty:

Faculty Name:	Department	Email Address	Office Phone Number

1. Describe the specific teaching and learning issues being addressed by the proposal.

Numerous topics in multivariable calculus involve three-dimensional geometry objects, e.g., points, vectors, lines, curves, planes, surfaces, and solids. Students often find these objects difficult to visualize, partially owing to the limitations inherent to the traditional 2-dimensional media, such as whiteboards or pages in textbooks.

We proposed to develop a collection of simple but effective modules which would allow the students to explore 3D objects in an interactive fashion (including zoom, rotate, pan viewport controls). The technology we proposed to use will ensure accessibility of these modules using a web browser on a wide variety of platforms, including smartphones.

2. Describe the revised specific teaching and learning issues being addressed by the proposal (if applicable):

As the development was underway, we decided to include in the project some modules pertaining to the Introductory Linear Algebra course. These modules illustrate various possible configurations of solution sets of systems of three equations in three unknowns.

3. Describe the development activities involved addressing the learning or teaching issue.

A total of fifty modules have been developed

- 14 modules illustrating Triple Integrals in Rectangular Coordinates,
- 11 modules illustrating Triple Integrals in Cylindrical and Spherical Coordinates,
- 10 modules illustrating the Divergence Theorem,
- 10 modules illustrating Stokes' Theorem, and
- 5 modules illustrating Systems of 3 Linear Equations in 3 Unknowns.

The modules were deployed to students via

- Handouts that contained QR codes to be scanned (and URL hyperlinks in PDF versions),
- QR codes with module hyperlinks being displayed using the overhead projector in the classroom,
- Physical bookmarks distributed to students, containing QR codes with module hyperlinks.

4. Describe the learning outcomes attained by the project.

In Spring 2015 section of MATH 312 (Calculus III) taught by Dr. Bogacki, a survey was conducted at the end of the semester, asking students whether they used the 3D illustrations and, if so, how helpful they found them. Of the 23 students who responded (out of 33 enrolled),

- 7 did not use the illustrations (when asked for reasons, some said they did not need help with 3D visualization),
- 16 did use them; of those,
    - 6 found them very helpful,
    - 10 found them mildly helpful,
    - No student selected “not helpful” as their response.

In addition to conducting the survey, Dr. Bogacki attempted to compare student performance on the final exam questions affected by the project (divergence theorem and Stokes’ theorem questions) before and after the project was implemented. While some improvement has been observed, it does not appear to be statistically significant (partially due to the small data sample).

5. Describe unexpected outcomes, if any.

The survey mentioned in #4 also asked students which platform (smartphone, tablet, or PC) they used to access the 3D models. Of the 16 students

- 7 used smartphones and PCs,
- 5 used smartphones only,
- 2 used PCs only,
- 1 used smartphones and tablets, and
- 1 used tablets only.

It was somewhat surprising to see that the tablets are used by very few students (but it was not surprising to see so many students use their smartphones.)

6. Describe the impact of the completed project on your colleagues, department, college, or community.

Dr. Bogacki communicates his work on this project and its outcomes will colleagues in Department of Mathematics and Statistics. Additionally, he delivered a contributed presentation on this undertaking during the Twenty-sixth International Conference on Technology in Collegiate Mathematics, held in March 2014 in San Antonio, Texas.

7. Describe how the project can be a model, template, or prototype for use by other instructors.

A template file has been created which should enable other instructors to use a similar approach to create 3D visualizations involving other parametric surfaces and curves. The file is available from the page
http://www.math.odu.edu/~bogacki/math312/figappendix/ .

8. Describe the technology used to help address the issues described in the proposal.

The technology solution we adopted for this project is based on the three.js (http://threejs.org) - a JavaScript-based library which offers the following key advantages:

- Three.js supports multiple hardware/software platforms.
    - It supports the HTML5 canvas element for displaying and interacting with 3D models (technically, it uses the two-dimensional canvas, and mathematically maps the 3D contents to them). A number of modern web browsers properly support canvas - this includes many browsers on smartphones and tablets running Android and iOS systems.
    - On hardware platforms and browsers supporting the WebGL technology, three.js can automatically render in that mode (instead of canvas), taking advantage of the Graphic Processing Unit (GPU) if present on the user's system. The resulting 3D model being displayed tends to be of much higher quality, and the user interaction is greatly improved (faster response time).
    - Other rendering modes (e.g., SVG) can also be used in three.js, but currently we opted to focus on canvas and WebGL only.

  When developing 3D contents (model, interaction) using Three.js library, one does not have to develop for a specific rendering mode (canvas vs. WebGL) - instead, the appropriate rendering mode can be chosen automatically according to the capabilities of the specific user's device and browser. Furthermore, this means that any contents developed under this project should be easily scalable to newer display technologies (by incorporating additional renderer modules).
- Three.js library is free to use and modify (under the MIT license).

Of particular concern was the touch interface compatibility – while three.js library comes equipped with “orbit control” and “trackball control” modules, neither of them performed completely satisfactorily on smartphone and tablet touch devices, and had to be extensively modified. The outcome of the current development combines these modified interface modules.

9. Describe products, if any, that are a result of the project.

Currently, the modules that have been built under this project can be accessed from the page
http://www.math.odu.edu/~bogacki/math312/figappendix/
However, these modules should not be considered a finished product yet, partially owing to the technical aspects of the touch interface interactivity, which continue to be refined. On the other hand, the modules have been successfully class-tested, and will be shared with interested colleagues.

10. Describe the future plans for this project, if any.

Primarily, Dr. Bogacki will continue to work on further modifying OrbitControls.js and/or TrackballControls.js code or possibly pursue an alternative to these modules to improve the touch interface interactivity. Once this is accomplished, other possible plans include

- equipping 3D models with additional interactive functionality (beyond the viewpoint control) to allow students to select specific portions of the model (e.g., by clicking on one of the surfaces surrounding a solid),
- enabling the students to construct their own 3D models on their smartphones by entering mathematical equations describing them, as well as possibly collaborating on models with other students and/or sharing them with the instructor,
- incorporating interactive animations of 3D models (e.g., an animated motion of an object along a given trajectory in space can be displayed, while allowing the student to change the viewpoint),
- designing clicker-type activities where students can respond to questions asked by the instructor by interacting with 3D models on their smartphone/tablet screens.

11. Attach a financial report with updated Budget Plan Matrix.

Final Budget Matrix

			Source of Funds	
Budget Item (equipment, personnel, software, etc.)	**Qty**	**Total Cost**	**Amount from FIG**	**Amount from Other Source**
Summer 2014 stipend for Dr. Bogacki		$1500	$1500	